CONSERVATION OF VOLUME
Solution to the Problem

Calculate the volume of the two cylinders using the formula:

$$V = \pi r^2 h$$

where:

V = volume of cylinder
r = radius of the cylinder
h = height of the cylinder (either 8½ or 11 inches)

Assuming that the volume of the two cylinders, 1 and 2, are equal:

$$V_1 = V_2$$

Or

$$\pi r_1{}^2 h_1 = \pi r_2{}^2 h_2$$

Use the substitution:

$r = c/2\pi$ (c = circumference of the cylinder, either 8½ or 11 inches)

$$\frac{\pi c_1{}^2 h_1}{4\pi^2} = \frac{\pi c_2{}^2 h_2}{4\pi^2}$$

Cancel π and $4\pi^2$ from both sides of the equation to get

$$c_1{}^2 h_1 = c_2{}^2 h_2$$

For cylinder 1: $c_1 = 8\ ½$ inches
                       $h_1 = 11$ inches

For cylinder 2: $c_2 = 11$ inches
                       $h_2 = 8\ ½$ inches

Substitute the numerical values into the equation (but do not multiply numbers):

$$(8\ ½\ x\ 8\ ½)\ x\ 11 = (11\ x\ 11)\ x\ 8\ ½$$

Cancel between both sides to get:

$$8\ ½ \neq 11$$

This shows that the volumes of the cylinders are not equal but are proportional to their circumferences.